Meritocracy and the Inheritance of Advantage

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Summary

Introduction *(1/2)*

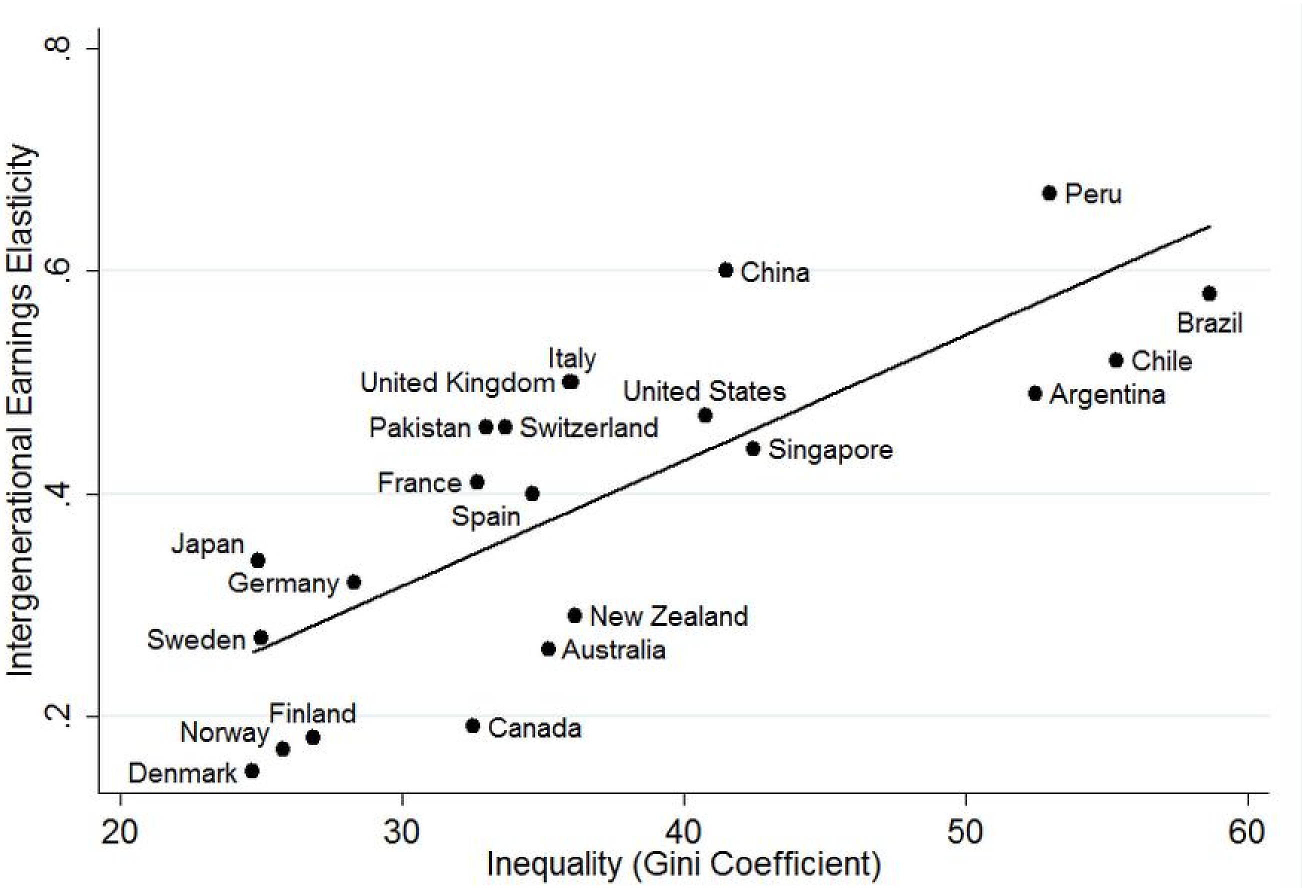
> *"We know all men are not created equal in the sense some people would have us believe - Some people are smarter than others, some people have more opportunity because they're born with it, some men make more money than others, some ladies make better cakes than others - some men are born gifted beyond the normal scope of most men."*
>
> *Atticus Finch, To Kill a Mockingbird (Harper Lee, 1960)*

- Michael Young's 1958 dystopia: **The Rise of Meritocracy 1870-2033**

- Organize our thoughts on
  - inequality, equality of opportunity, meritocracy.

- Model process by which some have more *opportunity* than others.
  - "beyond" capital market imperfections.

- Big negative correlation across countries Inequality-Mobility: Corak's "Great Gatsby" Curve.



Our Points

1. **Nurture, Investment and Statistical discrimination**
    - Family Income fosters Human Capital:
        - Capital market imperfections.
        - Provision of private education, etc...
    - Observable (with noise) whether the parents of kids are rich.
    - Thus, you can use the income of the parents to infer the talent of kids.
        - Statistical Discrimination.
            - Coate and Loury'93. Norman'03
    - More inequality implies more differences in talent between the children of rich and poor.
    - Thus, more statistical discrimination.
    - Which feeds back into:
        - Further income inequalities,
        - Lower intergenerational mobility.

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     - Further income inequalities,
     - Lower intergenerational mobility.

2. **Perverse GE effects of meritocracy**:
   - If firms are better at judging talent of individuals, more inequality.
   - which feeds back into more statistical discrimination...
   - More inequality and less mobility

Plan of Talk

- Human Capital Investment
- Pricing Merit and Advantage
- Advantages: Comparative Statics
- Meritocracy: Comparative Statics
- Both...
- Equilibrium
- ... "calibration" wishes
- Conclusions

- Families enjoy income Y_t^i
- Consume a bit (C_t^i), invest in their children education a bit (X_t^i)
- This investment helps determine children's human capital (H_{t+1}^i)
- The income of the children depends both on their human capital **and their parental income.**
  - This is endogenous to the model. A big part of our contribution
  - ... but so far take it as given.

Advantage:
Effect of parental income beyond human capital accumulation.

Meritocracy:
Effect of Human Capital beyond parental income.

$$W\left(Y_t^i\right) = \max_{X_t^i} \left\{ \ln C_t^i + \frac{1}{1+\delta} EW\left(Y_{t+1}^i | Y_t^i, X_t^i\right) \right\} \tag{1}$$

$s.t.$

$$C_t^i = Y_t^i - X_t^i; \quad X_t^i \geq 0 \tag{2}$$

$$H_{t+1}^i \sim G\left(X_t^i\right) \tag{3}$$

$$Y_{t+1}^i \sim F\left(H_{t+1}^i, Y_t^i\right) \tag{4}$$

- We assume that human capital accumulation is:

$$H_{t+1}^i = Z\left(X_t^i\right)^\alpha e^{\tilde{\omega}_{t+1}^i}; \quad \tilde{\omega}_{t+1}^i \sim N\left(-\frac{V_\omega}{2}, V_\omega\right) \tag{5}$$

- and so far we assume that the income process is:

$$Y_{t+1}^i = e^{\gamma_0}\left(Y_t^i\right)^{\gamma_1}\left(X_t^i\right)^{\gamma_2} e^{\varepsilon_{t+1}^i} \tag{6}$$

γ_0, γ_1 and γ_2 and the distribution of ε_{t+1}^i are endogenous to the model, but exogenous to the individual.

We will find the equilibrium values of γ_0, γ_1 and γ_2 and the distribution of ε_{t+1}^i

The solution of the maximization problem in equation 1 requires that investment in education is a fixed proportion of the individual's income: $X_t^i = \lambda Y_t^i$, with:

$$\lambda = \frac{\gamma_2}{1 + \delta - \gamma_1} \tag{7}$$

The value function of agents is $W(Y) = A + B \ln Y$, with

$$A = \frac{\bar{\varepsilon} + \gamma_0 + \ln\left[(1+\delta) - (\gamma_1 + \gamma_2)\right]^{[(1+\delta)-(\gamma_1+\gamma_2)]} + \ln \frac{\gamma_2^{\gamma_2}}{[(1+\delta)-\gamma_1]^{[(1+\delta)-\gamma_1]}}}{\left[(1+\delta) - (\gamma_1 + \gamma_2)\right] \frac{\delta}{1+\delta}} \tag{8}$$

$$B = \frac{(1+\delta)}{(1+\delta) - (\gamma_1 + \gamma_2)} \tag{9}$$

Log human capital is a linear function of log parental income. The elasticity is an exogenous parameter α, while the constant depends on the investment rate λ :

$$h_{t+1}^i = \ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha y_t^i + \omega_{t+1}^i \tag{10}$$

where $\omega_{t+1}^i \sim N(0, V_\omega)$ is iid noise and h_{t+1}^i and y_t^i represent the log of the child's human capital and the log of parental income respectively.

- Firms hire workers and produce biscuits. Assign them to roles related to their H.
- Workers with more H make more biscuits
- Putting workers in a role for wich they are not suited decreases biscuit production: **missallocation**

$$D^i_{t+1} = \exp\left\{h^i_{t+1} - \frac{\theta}{2}\left(h^i_{t+1} - E(h^i_{t+1}|\Omega^i_{t+1})\right)^2\right\}; \qquad \theta \in R^+ \qquad (11)$$
$$Y^i_{t+1} = E(D^i_{t+1}|\Omega^i_{t+1})$$

- The problem of firms is that they do not know h of their workers
- They only have some information on it

Information Available for pricing h

$$\Omega^i_{t+1} = \left\{a^i_{t+1}, m^i_{t+1}, \mu_y, V_y\right\}$$

(1) a public signal on the parental income of agent i: a^i_{t+1};

(2) a public signal on the human capital of agent i: m^i_{t+1};

(3) the distribution of income in the parent's generation; $y^i_t \sim N\left(\mu_{yt}, V_{yt}\right)$

(4) the equation of accumulation of human capital 10

- a^i_{t+1}, is a publicly available signal on parental income such that:

$$a^i_{t+1} = y^i_t + \varepsilon^{ia}_{t+1}$$ $\varepsilon^{ia}_{t+1} \sim N(0, V_a)$ is iid noise.

- m^i_{t+1}, is a publicly available signal on human capital such that:

$$m^i_{t+1} = h^i_{t+1} + \varepsilon^{im}_{t+1}$$ $\varepsilon^{im}_{t+1} \sim N(0, V_m)$ is iid noise.

- The precision of the signals is our measure of the (exogenous) prevalence of meritocracy and advantage.

Pricing Merit and Advantage *(3/6)*

Given the process of human capital accumulation and the information set, the posterior of the log of human capital is given by:

$$h^i_{t+1}|\Omega^i_{t+1} \sim N\left(\mu_{h^i_{t+1}|\Omega^i_{t+1}}, V_{h^i_{t+1}|\Omega^i_{t+1}}\right)$$

with

$$\mu_{h^i_{t+1}|\Omega^i_{t+1}} = \beta_m m^i_{t+1} + (1-\beta_m)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha\beta_a a^i_{t+1} + \alpha(1-\beta_a)\mu_y\right]$$

$$V_{h^i_{t+1}|\Omega^i_{t+1}} = \beta_m V_m$$

$$\beta_a = \frac{V_y}{V_y + V_a}$$

$$\beta_m = \frac{\alpha^2\beta_a V_a + V_\omega}{\alpha^2\beta_a V_a + V_\omega + V_m}$$

The posterior belief about the human capital of individual i at time $t+1$ is a stochastic function of their observable signals, a^i_{t+1} and m^i_{t+1}. The variance in those beliefs is actually constant across individuals and (in steady state) across time.

Given the posterior belief about the log of human capital follows a normal distribution

$$h_{t+1}^i | \Omega_{t+1}^i \sim N\left(\mu_{h_{t+1}^i|\Omega_{t+1}^i}, V_{h_{t+1}^i|\Omega_{t+1}^i}\right)$$

and income is given by equation 11, it follows that:

$$Y_{t+1}^i = \frac{1}{\sqrt{1+\theta V_{h_{t+1}^i|\Omega_{t+1}^i}}} \exp\left\{\frac{1}{2}\left(\frac{V_{h_{t+1}^i|\Omega_{t+1}^i}}{1+\theta V_{h_{t+1}^i|\Omega_{t+1}^i}}\right)\right\} \exp\left\{\mu_{h_{t+1}^i|\Omega_{t+1}^i}\right\} \tag{12}$$

By taking logs and substituting we can find the log income of individual i with signals a_{t+1}^i and m_{t+1}^i:

$$y_{t+1}^i = (1-\beta_m)\left[\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha(1-\beta_a)\mu_y\right] - \frac{1}{2}\left[\ln(1+\theta\beta_m V_m) - \frac{\beta_m V_m}{1+\theta\beta_m V_m}\right] + \alpha\beta_a(1-\beta_m)a_{t+1}^i + \beta_m m_{t+1}^i \tag{13}$$

- β_m is the weight given to the signal of an agent's human capital when determining her income
- $\widehat{\beta}_a = \alpha\beta_a(1-\beta_m)$ is the weight given to the signal on parent's income

Given that a_{t+1}^i and m_{t+1}^i are both stochastic functions of y_t^i we can write the law of motion of the log of income:

$$y_{t+1}^i = \ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} + \alpha(1-\beta_a)(1-\beta_m)\mu_y - \frac{1}{2}\left[\ln(1+\theta\beta_m V_m) - \frac{\beta_m V_m}{1+\theta\beta_m V_m}\right]$$
$$+\alpha\left[\beta_a(1-\beta_m)+\beta_m\right]y_t^i + \alpha\beta_a(1-\beta_m)\varepsilon_{t+1}^{ia} + \beta_m\left(\varepsilon_{t+1}^{im}+\omega_{t+1}^i\right) \quad (14)$$

The intergenerational income elasticity is $\rho = \alpha\left[\beta_a(1-\beta_m)+\beta_m\right]$

the law of motion of the variance of log income is:

$$V_{y_{t+1}} = \alpha^2\left[\beta_a(1-\beta_m)+\beta_m\right]V_{y_t} + \beta_m V_\omega \quad (15)$$

Feed back mechanism

- $\uparrow \beta_a$ or β_m, $\Rightarrow \uparrow V_{y_{t+1}}$
- This interacts with the equations determining β_a and β_m (how much firms discriminate according to background and perceived merit)
- β_a and β_m depend on V_{y_t}

Steady State.
There exists a unique steady state which is globally stable. In the steady state log income is normally distributed with variance being characterized by the (unique) solution of the following system of equations:

$$V_y = \frac{\beta_m V_\omega}{1 - \alpha^2 \left[\beta_a \left(1 - \beta_m\right) + \beta_m\right]} \tag{16}$$

$$\beta_a = \frac{V_y}{V_y + V_a} \tag{17}$$

$$\beta_m = \frac{\alpha^2 \ \beta_a \ V_a + V_\omega}{\alpha^2 \ \beta_a \ V_a + V_\omega + V_m} \tag{18}$$

The steady state mean of log income and intergenerational correlation of income are given by:

$$\mu_y = \frac{\ln Z + \alpha \ln \lambda - \frac{V_\omega}{2} - \frac{1}{2}\left[\ln\left(1 + \theta\beta_m V_m\right) - \frac{\beta_m V_m}{1+\theta\beta_m V_m}\right]}{1 - \alpha} \tag{19}$$

$$\rho = \alpha \left[\beta_a \left(1 - \beta_m\right) + \beta_m\right] \tag{20}$$

Advantages *(1/8)*

- Human Capital is a function of parental income and luck
  - (so far) Exogenous linear function determining HK (talent):

$$h_t^i = \widehat{\phi} + \alpha y_{t-1}^i + \omega_t^i$$

  - h_t^i is log human capital of agent i
  - y_{t-1}^i is log income of her parent,
  - ω_t^i is iid noise with variance V_ω which is constant across time and individuals,
  - $\alpha < 1$ is an exogenous parameter.

- Talent is not perfectly observed.
  - Priors
  - Public signal on parental income

$$a_t^i = y_{t-1}^i + \varepsilon_t^{ia}$$

  - $\varepsilon^{ia} \sim N(0, V_a)$ is iid noise.

- Agents paid their expected human capital given the available information, plus some noise

$$Y_t^i = E(H_t^i|\Omega_t^i).\exp\left\{u_t^i\right\}$$

  - $u \sim N(0, V_u)$ and is iid.
  - Ω_t^i is the set of available information.

$$\Omega_t^i = \left\{a_t^i, \mu_{y_{t-1}}, V_{y_{t-1}}\right\}$$

- $\left\{\mu_{y_{t-1}}, V_{y_{t-1}}\right\}$ determines the distribution of incomes at t via the interpretation that the market gives to the signals.
  - A steady state is reached when these distributions are identical.

- Interesting bit: income distribution determines interpretation market gives to signals, which determines distribution...

Advantages *(3/8)*

- Past income determines today's ability (nurture).
  - You want to use info on parents to guess i's talent
  - but how much you use it depends on how much you know

- If talent depends on parents income, **more inequality** means that you **care more about the signal**
  - Which makes people **incomes more different**, because they have an **extra meaningful dimension** in which they differ.

- If $\downarrow \sigma_y^2$, you do not care about the signal because
  - They are similar, anyway
  - But also because the signal is KNOWN to be UNINFORMATIVE

- If $\uparrow \sigma_y^2$, you want to use the signal because
  - They are different
  - And the signal IS INFORMATIVE

Advantages *(4/8)*

More inequality

$\Downarrow$ more dispersion of talent $\Downarrow$ more value on the signals, $\uparrow \beta_1$ $\Downarrow$

$\Downarrow$ more dispersion of the signals $\Downarrow$

More dispersion of incomes

The existence of people that are rich
gives advantages to their children
that go beyond their abilities.

- Posterior of parental income:

$$y_{t-1}^{i}|a_{t}^{i} \;\sim\; N\left(E\left[y_{t-1}^{i}|a_{t}^{i}\right], V\left[y_{t-1}^{i}|a_{t}^{i}\right]\right)$$

$$E\left[y_{t-1}^{i}|a_{t}^{i}\right] = \beta_{a_t}\; a_{t}^{i} + (1-\beta_{a_t})\;\mu_{y_{t-1}}; \quad V\left[y_{t-1}^{i}|a_{t}^{i}\right] = \beta_{a_t}\; V_a$$

$$\beta_{a_t} = \frac{V_{y_{t-1}}}{V_{y_{t-1}} + V_a}$$

- Posterior on child's talent:

$$h_{t}^{i}|a_{t}^{i} \;\sim\; N\left(E\left[h_{t}^{i}|a_{t}^{i}\right], V\left[h_{t}^{i}|a_{t}^{i}\right]\right)$$

$$E\left[h_{t}^{i}|a_{t}^{i}\right] = \widehat{\phi} + \alpha\left[\beta_{a_t}\; a_{t}^{i} + (1-\beta_{a_t})\;\mu_{y_{t-1}}\right]$$

$$V\left[h_{t}^{i}|a_{t}^{i}\right] = \alpha^{2}\;\beta_{a_t}\; V_a + V_{\omega}$$

- β_a measures advantage of background independent of talent

Advantages *(6/8)*

- Process of log income:

$$y_t^i = \phi + \frac{\alpha^2 \ \beta_{a_t} \ V_a}{2} + \alpha \left(1 - \beta_{a_t}\right) \ \mu_{y_{t-1}} + \alpha\beta_{a_t} \ a_t^i + u_t^i$$

  - Becker-Tomes style: $\rho_t = \alpha\beta_{a_t}$

$$y_t^i = \phi + \frac{\alpha^2 \ \beta_{a_t} \ V_a}{2} + \alpha \left(1 - \beta_{a_t}\right) \ \mu_{y_{t-1}} + \alpha\beta_{a_t} \ y_{t-1}^i + \alpha\beta_{a_t} \ \varepsilon_t^{ai} + u_t^i$$

- Law of Motion of inequality:

$$V_{y_t} = \alpha^2 \beta_{a_t} V_{y_{t-1}} \ + V_u$$

$$V_{y_t} = \alpha^2 \beta_{a_t} V_{y_{t-1}} + V_u; \qquad \beta_{a_t} = \frac{V_{y_{t-1}}}{V_{y_{t-1}} + V_a}$$

- If $\alpha \in (0,1)$ and $\Omega_t^i = \left\{a_t^i, \mu_{y_{t-1}}, V_{y_{t-1}}\right\}$, there exists a unique steady state, which is globally stable. In the steady state log income is normally distributed with mean and variances being characterized by the (unique) solution of the following system of equations:

$$\mu_y = \frac{\phi + \frac{\alpha^2 \; \beta_a \; V_a}{2}}{1 - \alpha}$$

$$V_y = \frac{V_u}{1 - \alpha^2 \beta_a}$$

$$\beta_a = \frac{V_y}{V_y + V_a}$$

- **Multiplier:** The full effect of an exogenous change in the parameters V_u or α is greater than the partial effect due to the feed back from income inequality to discrimination

$$\frac{dV_y}{dV_u} > \frac{\partial V_y}{\partial V_u} > 0; \quad \frac{d\beta_a}{dV_u} > \frac{\partial \beta_a}{\partial V_u} > 0$$
$$\frac{dV_y}{d\alpha} > \frac{\partial V_y}{\partial \alpha} > 0; \quad \frac{d\beta_a}{d\alpha} > \frac{\partial \beta_a}{\partial \alpha} > 0$$

- **Advantages:** In steady state V_y, ρ and β_a are all decreasing in V_a.

- Not obvious. More noise, less inequality.
  - given β_a, more noise, less variance next period.
  - It is the endogenous adjustment of β_a

- Now: signal on talent (and forget advantage)

$$m_t^i = h_t^i + \varepsilon_t^{im}$$

with iid noise $\varepsilon^{im} \sim N\left(0, V_m\right)$. **Meritocracy: low V_m**

- Info: $\Omega_t^i = \left\{m_t^i, \mu_{y_{t-1}}, V_{y_{t-1}}\right\}$

- Assume (from now on) $V_u = 0$.
  - Luck only in talent accumulation and signals.

- Then, unconditional dispersion of talent: $V_{h,t} = \alpha^2 V_{y,t-1} + V_\omega$.
  - Posterior:

$$h_t^i | m_t^i \sim N\left(\left(1 - \beta_{m,t}\right)\left(\hat{\phi} + \alpha\mu_{y_{t-1}}\right) + \beta_{m,t} m_t^i \;\; , \;\; \beta_{m,t} V_m\right)$$

$$\beta_{m,t} = \frac{V_{h,t}}{V_{h,t} + V_m}$$

The Curse of Meritocracy *(2/6)*

$$Y_t^i = E\left(H_t^i | m_t^i\right) = \exp\left\{E\left[h_t^i | m_t^i\right] + \frac{V\left[h_t^i | m_t^i\right]}{2}\right\}$$

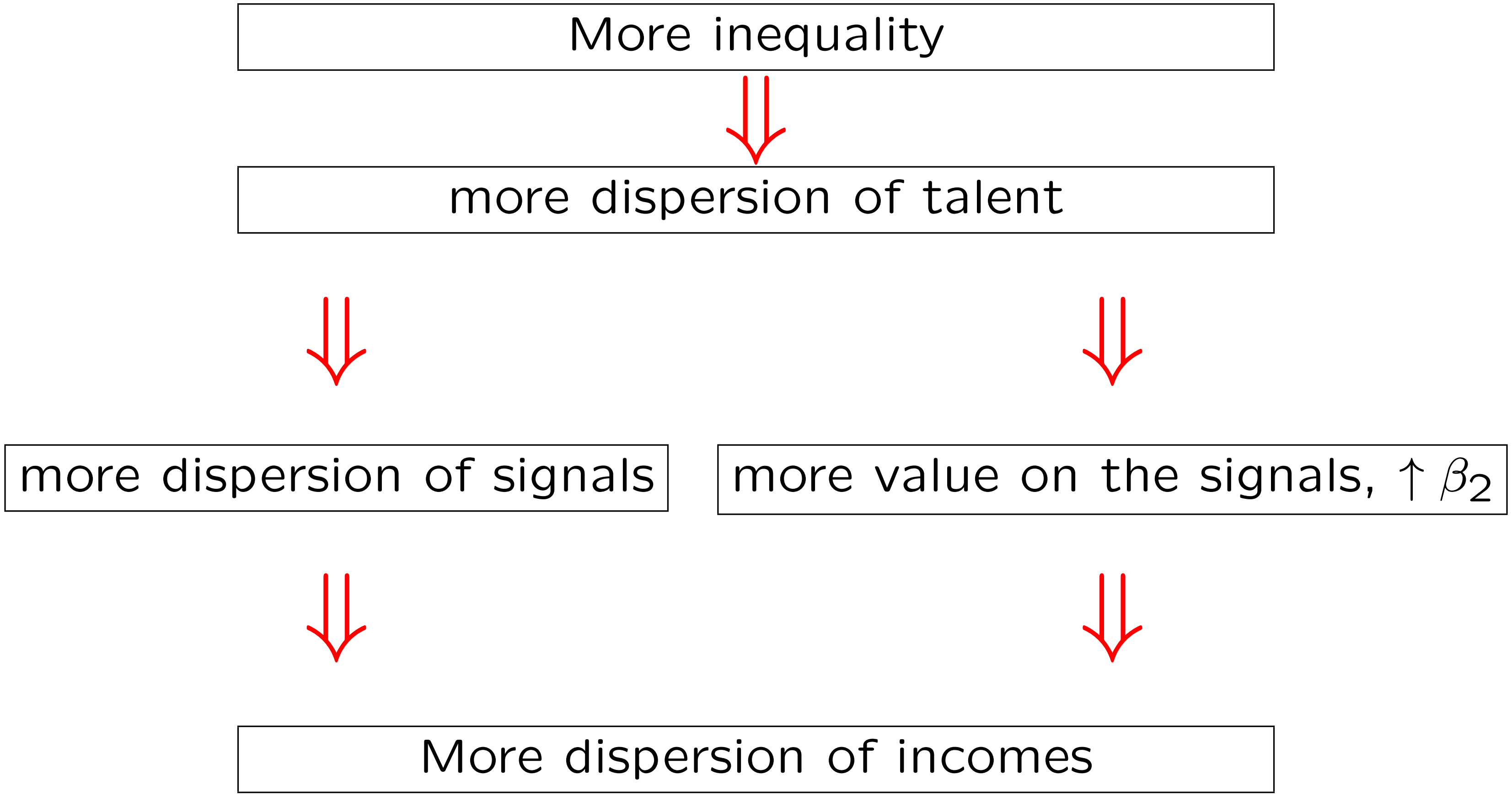
$$y_t^i = \left[\left(1 - \beta_{m,t}\right)\left(\widehat{\phi} + \alpha\mu_{y_{t-1}}\right) + \frac{1}{2}\beta_{m,t}V_m\right] + \beta_{m,t}\ m_t^i$$

- Law of motion:

$$V_{h,t+1} = \alpha^2\ \beta_{m,t}\ V_{h,t} + V_\omega; \quad \beta_{m,t} = \frac{V_{h,t}}{V_{h,t} + V_m}$$

The Curse of Meritocracy *(3/6)*

Feed back



The Curse of Meritocracy *(4/6)*

- Steady state intergenerational correlation of incomes is given by,

$$\rho_{y,y_{-1}} = \alpha\beta$$

$$\downarrow V_m \quad \Rightarrow \quad \uparrow \beta_2 \quad \Rightarrow \quad \uparrow \rho_{y,y_{-1}}$$

Increase meritocracy ⇒ firms better at picking talent ⇒ talented likely to have rich parents ⇒ Decrease mobility

- An INCREASE in meritocracy leads to a DECREASE in mobility
  - via a general equilibrium effect:
  - Increase of inequality

- If $\alpha \in (0,1)$ and $\Omega_t^i = \left\{m_t^i, \mu_{y_{t-1}}, V_{y_{t-1}}\right\}$, there exists a unique steady state, which is globally stable. In the steady state log income is normally distributed with mean and variances being characterized by the (unique) solution of the following system of equations:

$$V_h = \frac{V_\omega}{1 - \alpha^2 \beta_m}$$
$$\beta_m = \frac{V_h}{V_h + V_m}$$

- The intergenerational correlation of log income is $\rho = \alpha\beta_m$, while its variance is $V_y = \frac{\beta_m V_\omega}{1-\alpha^2\beta_m}$

- The steady state values of V_h, V_y, ρ and β_m are all decreasing in V_m.

The Curse of Meritocracy *(6/6)*

- Meritocracy decreases mobility and increases inequality
  - Not only via inheritance, also by increasing the value of the signals...
  - Fedd back to itself.

- Meritocracy is not that different from advantages.
  - More inequality,
  - ... more value of info,
  - ... more inequality

Advantage and Meritocracy *(1/7)*

- $\Omega_t^i = \left\{a_t^i,\ m_t^i,\ \mu_{y_{t-1}},\ V_{y_{t-1}}\right\}$, $Y_t^i = E\left(H_t^i|\Omega_t^i\right)$

- Now the two signals may feed back to each other via the degree of inequality.

$$y^i_{t-1}|a^i_t \;\sim\; N\left(\beta_{a_t}\; a^i_t + (1-\beta_{a_t})\;\; \mu_{y_{t-1}}, \beta_{a_t}\; V_a\right);\; \beta_{a_t} = \frac{V_{y_{t-1}}}{V_{y_{t-1}}+V_a}$$

$$h^i_t|a^i_t \;\sim\; N\left(E\left[h^i_t|a^i_t\right], V\left[h^i_t|a^i_t\right]\right)$$

$$E\left[h^i_t|a^i_t\right] = \widehat{\phi} + \alpha\left[\beta_{a_t}\; a^i_t + (1-\beta_{a_t})\;\; \mu_{y_{t-1}}\right]$$

$$V\left[h^i_t|a^i_t\right] = \alpha^2\; \beta_{a_t}\; V_a + V_\omega$$

$$h^i_t|a^i_t, m^i_t \;\sim\; N\left(\beta_{m_t} m^i_t + (1-\beta_{m_t})\, E\left[h^i_t|a^i_t\right], \beta_{m_t} V_m\right);\; \beta_{m_t} = \frac{V\left[h^i_t|a^i_t\right]}{V\left[h^i_t|a^i_t\right]+V_m}$$

$$Y^i_t = E\left(H^i_t|a^i_t, m^i_t\right) = \exp\left\{E\left[h^i_t|a^i_t, m^i_t\right] + \frac{V\left[h^i_t|a^i_t, m^i_t\right]}{2}\right\}$$

Advantage and Meritocracy *(3/7)*

$$y_t^i = \left[(1-\beta_{m_t}) \left\{ \widehat{\phi} + \alpha \left(1-\beta_{a_t}\right) \ \mu_{y_{t-1}} \right\} + \frac{1}{2}\beta_{m_t} V_m \right] + \alpha\beta_{a_t} \left(1-\beta_{m_t}\right) \ a_t^i + \beta_{m_t} \, m_t^i$$

- β_{m_t}: weight given to the signal of an agent's human capital when determining her income

- $\widehat{\beta}_{a_t} = \alpha\beta_{a_t}\left(1-\beta_{m_t}\right)$: weight of signal on parent's income.

- There exists a certain trade-off between merit and advantages: β_m directly decreases $\widehat{\beta}_a$,
  - Not obvious, both β_a and β_m are endogenous.

- Law of motion of the variance of log income:

$$V_{y_t} = \alpha^2 \left[\beta_{a_t}\left(1-\beta_{m_t}\right)+\beta_{m_t}\right] V_{y_{t-1}} + \beta_{m_t} V_\omega$$
$$\beta_{a_t} = \frac{V_{y_{t-1}}}{V_{y_{t-1}}+V_a}; \quad \beta_{m_t} = \frac{\alpha^2\ \beta_{a_t}\ V_a + V_\omega}{\alpha^2\ \beta_{a_t}\ V_a + V_\omega + V_m}$$

- Given a process of human capital accumulation $h_t^i = \widehat{\phi} + \alpha y_{t-1}^i + \omega_t^i$ with $\alpha \in (0,1)$, and $\Omega_t^i = \left\{a_t^i, m_t^i, \mu_{y_{t-1}}, V_{y_{t-1}}\right\}$, there exists a unique steady state, which is globally stable. In the steady state log income is normally distributed with mean and variances being characterized by the (unique) solution of the following system of equations:

$$\mu_y = \frac{\widehat{\phi} + \frac{1}{2}\beta_m V_m}{1-\alpha}; \quad V_y = \frac{\beta_m V_\omega}{1-\alpha^2\left[\beta_a\left(1-\beta_m\right)+\beta_m\right]}$$
$$\beta_a = \frac{V_y}{V_y+V_a}; \quad \beta_m = \frac{\alpha^2\ \beta_a\ V_a + V_\omega}{\alpha^2\ \beta_a\ V_a + V_\omega + V_m}$$

- The intergenerational correlation of income is $\rho = \alpha\left[\beta_a\left(1-\beta_m\right)+\beta_m\right]$

More Advantages: Increase in the accuracy of the signal on background (a decrease of V_a) results, in steady state, in more inequality, greater persistence of income across generations, more discrimination based on perceptions of the background of an agent, and a smaller elasticity of income to the signal on ability:

$$\frac{dV_y}{dV_a} < 0; \quad \frac{d\rho}{dV_a} < 0; \quad \frac{d\beta_a}{dV_a} < 0; \quad \frac{d\widehat{\beta}_a}{dV_a} < 0; \quad \frac{d\beta_m}{dV_a} > 0$$

- $\downarrow V_a \Rightarrow \uparrow V_y \Rightarrow \uparrow \beta_a$.
  - more advantage, more ineq, more advantage
- $\downarrow V_a \Rightarrow \downarrow \beta_m$. "crowding-out"
  - more advantage, less meritocracy
  - Increase of unconditional variance of h...
  - But decrease of variance of h conditional on a. More advantage, less use of meritocracy signal
- $\downarrow V_a \Rightarrow \uparrow \rho$

More meritocracy

- An increase in the accuracy of the signal on ability (a decrease of V_m) results, in steady state, in more inequality, greater persistence of income across generations, a larger elasticity of income to the signal on ability and more weight given to the signal on the background when evaluating an agent's parental income:

$$\frac{dV_y}{dV_m} < 0; \quad \frac{d\rho}{dV_m} < 0; \quad \frac{d\beta_m}{dV_m} < 0; \quad \frac{d\beta_a}{dV_m} < 0$$

- Moreover, given a set of values for $\alpha \in (0,1)$ and $V_a \in R^+$ $(V_a < \infty)$, there exists a variance of the signal on ability $\widehat{V}_m$ such that $\left(0 < \widehat{V}_m < \infty\right)$

$$\text{If } V_m < \widehat{V}_m, \text{ then } \frac{d\widehat{\beta}_a}{dV_m} > 0$$

$$\text{If } V_m > \widehat{V}_m, \text{ then } \frac{d\widehat{\beta}_a}{dV_m} < 0$$

The value of $\widehat{\beta}_a$ is maximal if $V_m = \widehat{V}_m$.

More meritocracy

- $\downarrow V_m \Rightarrow \uparrow \beta_m \Rightarrow \uparrow V_y \Rightarrow V_h \Rightarrow \uparrow \beta_m \ldots$

- $\downarrow V_m \Rightarrow \uparrow V_y \Rightarrow \uparrow \beta_a$

- $\widehat{\beta}_a = \alpha \beta_a (1 - \beta_m)$ is more comlicated
  - Crowding out because $\uparrow \beta_m$
  - Increase because $\uparrow V_y$
  - meritocracy may increase advantages!

- Always: $\downarrow V_m \Rightarrow \uparrow \rho, \uparrow V_y$
  - **Meritocracy and Advantages are very similar monsters**

- A parental investment decision rule that is optimal
- Given a certain process of income determination as a function of prental income and human capital
- And this process to be the outcome of firm's taking decisions understanding the parents investment process.

Equilibrium. The equilibrium stochastic process of income as a function of parental income and investment is $Y_{t+1}^i = e^{\gamma_0} \left(Y_t^i\right)^{\gamma_1} \left(X_t^i\right)^{\gamma_2} e^{\varepsilon_{t+1}^i}$ with:

$$\gamma_0 = \ln Z + \alpha \left(1 - \beta_m\right) \left[\left(1 - \beta_a\right) \mu_y + \ln \lambda\right] - \frac{1}{2} \left[\ln \left(1 + \theta \beta_m V_m\right) - \frac{\beta_m V_m}{1 + \theta \beta_m V_m} + V_\omega\right]$$

$$\gamma_1 = \alpha \beta_a \left(1 - \beta_m\right)$$

$$\gamma_2 = \alpha \beta_m$$

$$\varepsilon_{t+1}^i = \alpha \beta_a \left(1 - \beta_m\right) \varepsilon_{t+1}^{ai} + \beta_m \left(\omega_{t+1}^i + \varepsilon_{t+1}^{mi}\right)$$

and, consequently, the equilibrium share of income invested in children's education is:

$$\lambda = \frac{\alpha \beta_m}{1 + \delta - \alpha \beta_a \left(1 - \beta_m\right)} \tag{21}$$

Equilibrium *(2/3)*

An increase in the accuracy of the human capital signal (a decrease of V_m) results, in steady state, in an increase in the proportion of income invested in education. An increase in the accuracy of the signal on background (a decrease of V_a) may increase or decrease investment. A sufficient condition for $\frac{d\lambda}{dV_a} < 0$ is:

$$\frac{V_m}{V_\omega}\left[\frac{1-\alpha^2\left[\beta_a\left(1-\beta_m\right)+\beta_m\right]}{\left[(1+\delta)-\alpha\beta_a\right]}\right] > 1$$

Equilibrium *(3/3)*

When misallocation is sufficiently costly (θ is above a threshold level), an increase in the accuracy of either signal will raise median income. The threshold value above which this occurs for the ability signal, $\bar{\theta}$, solves:

$$\frac{2\alpha}{\lambda}\frac{\frac{d\lambda}{dV_m}}{\left[\beta_m + V_m\frac{d\beta_m}{dV_m}\right]} = \frac{\left[\bar{\theta}\left(1+\bar{\theta}\beta_m V_m\right)-1\right]}{\left(1+\bar{\theta}\beta_m V_m\right)^2} \tag{22}$$

The threshold value above which this occurs for the background signal, $\hat{\theta}$, solves:

$$\frac{2a}{\lambda}\frac{\frac{d\lambda}{dV_a}}{V_m\frac{d\beta_m}{dV_a}} = \frac{\left[\hat{\theta}\left(1+\hat{\theta}\beta_m V_m\right)-1\right]}{\left(1+\hat{\theta}\beta_m V_m\right)^2} \tag{23}$$

Calibration

- Include Private and Public education

Calibration

- Include Private and Public education
  - esay

Calibration

- Include Private and Public education
  - esay
- Include redistribution

Calibration

- Include Private and Public education
  - esay

- Include redistribution
  - doable

Calibration

- Include Private and Public education
  - esay
- Include redistribution
  - doable
- ...

Calibration

- Include Private and Public education
  - esay
- Include redistribution
  - doable
- ... when I'm done I'll tell you
- Exercise: Variation across countries in V_m and V_a

Conclusion

- I learnt:
  - Advantages everywhere
  - Meritocracy is very much like advantages when looked from outside
  - Meritocracy decreases mobility and increases inequality
    - It may even increase advantages.
    - Happiness and income.
- To do
  - Calibration
  - Include Nature/Genetics/Assortative Mating
  - Effort?

Meritocracy and the Inheritance of Advantage

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